University of Sheffield

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ERMABI – Exploiting the Resilience of Masonry Arch Bridge Infrastructure

Fast-running analysis models for masonry arch bridges

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- Key concepts:
  - homogenization
  - parametric geometry
- Homogenized DLO formulation
- Numerical examples
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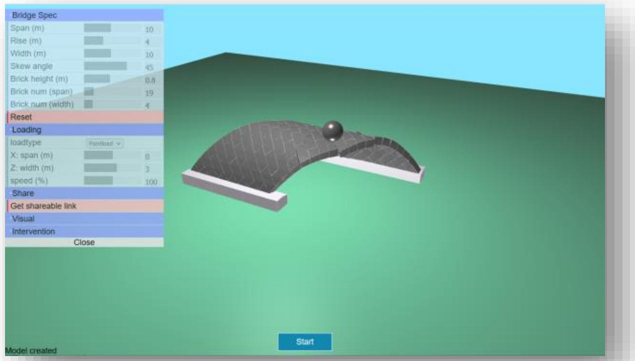
Background

Backgrounds

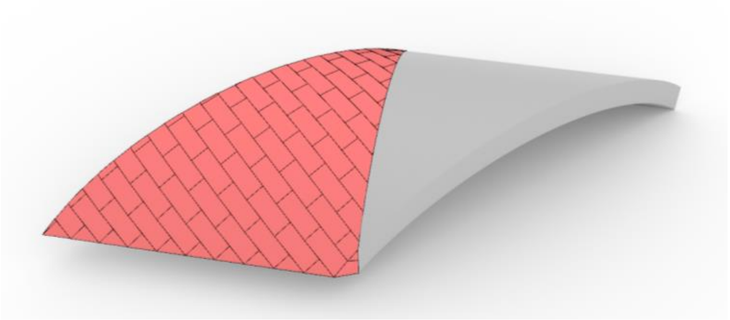
List of fast-running tools:

Rigid-block Analysis

Physics Engine

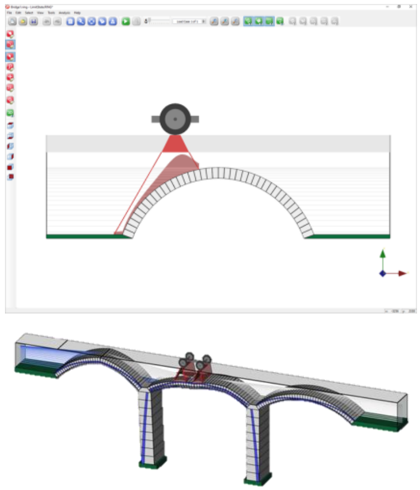


(D)iscontinuity (L)ayout (O)ptimization

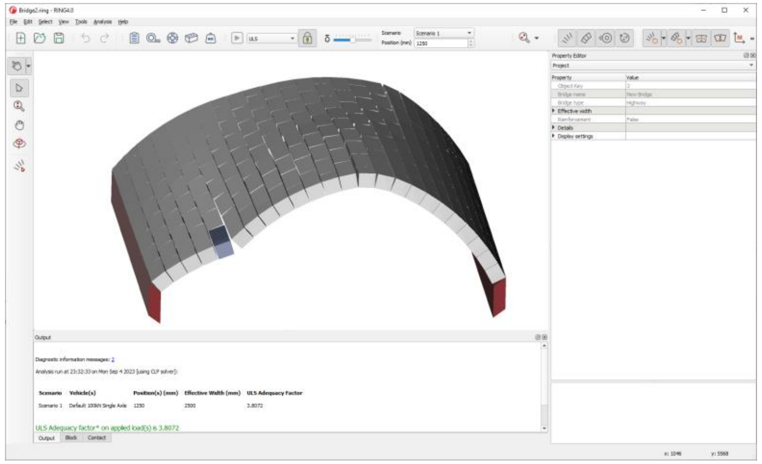


Rigid block analysis

- Simple, various tools are available
- Requires detailed geometry
- Various block interaction rules possible
- Possibility to identify local failure modes



LimitState:RING



LimitState:RING 3D
(under development)

Parametric modelling workflow

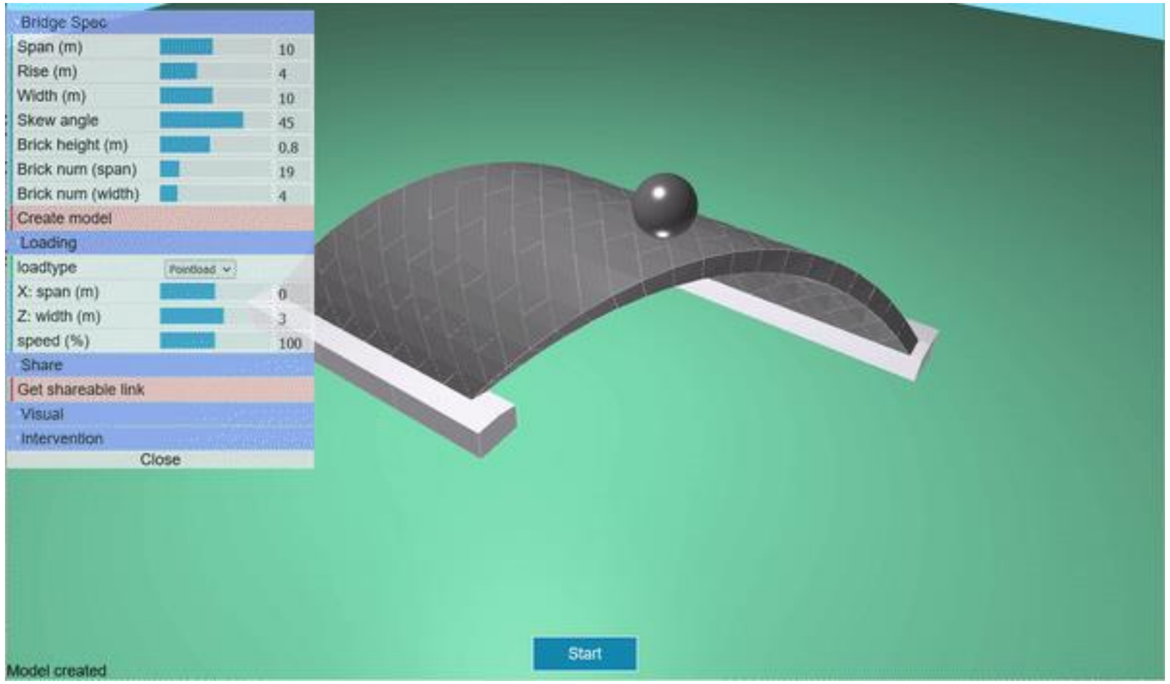
Physics engine

Web-app to simulate collapse behaviour of square or skew bridges:

- Fast: simulation in seconds/minutes
- Web-app: zero configuration
- Interactive: pause and intervene
- Versatile: freely impose arbitrary abutment block settlements

Cons:

- Qualitative results only
- Only masonry elements modelled

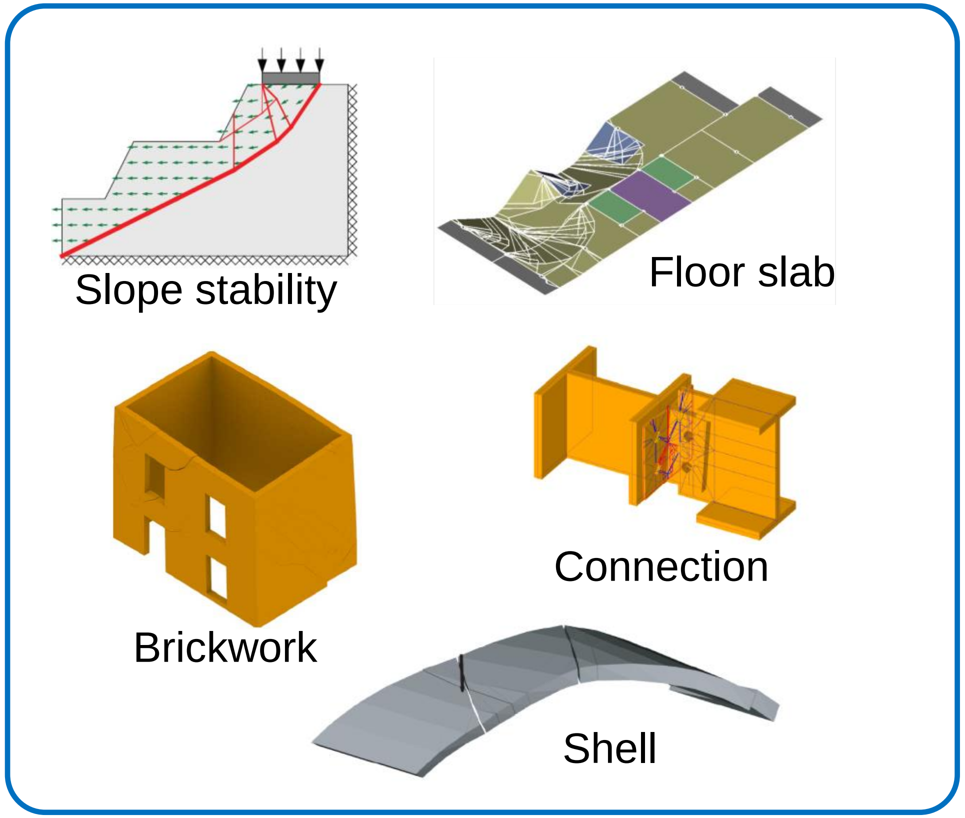


See https://ermabi.org (under development)

Discontinuity layout optimization (DLO)

Novel numerical process to identify the load carrying capacity of structures:

- Reliable: mathematics ensures highly accurate solutions
- Efficient: problems with >10,000,000 variables solvable on modern laptops.
- Opensource: shared Python scripts.
- Versatile: broad spectrum of applications



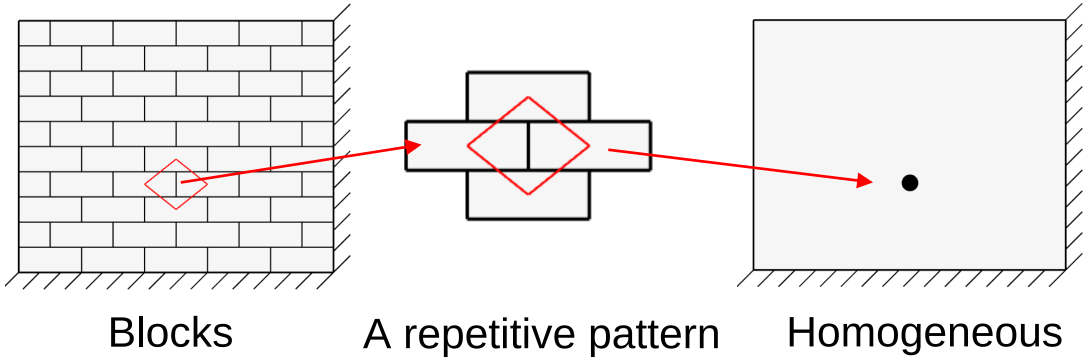
Cons:

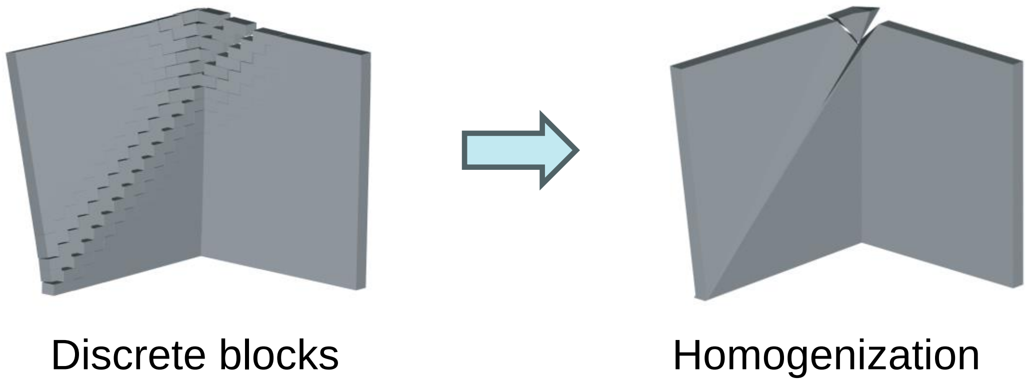
- Developments needed in order to apply to masonry arch bridges

Homogenized DLO

Modelling masonry blocks in DLO:

- Homogenization method
- Potentially complicated failure criteria, e.g., crushing behaviour
- System failure modes, insensitive to small geometry variations
- Effectively models infinitely small blocks; conservative results

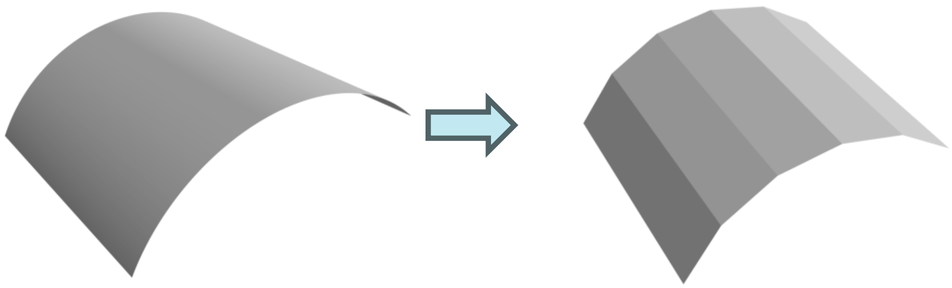




MA bridges – two DLO approaches

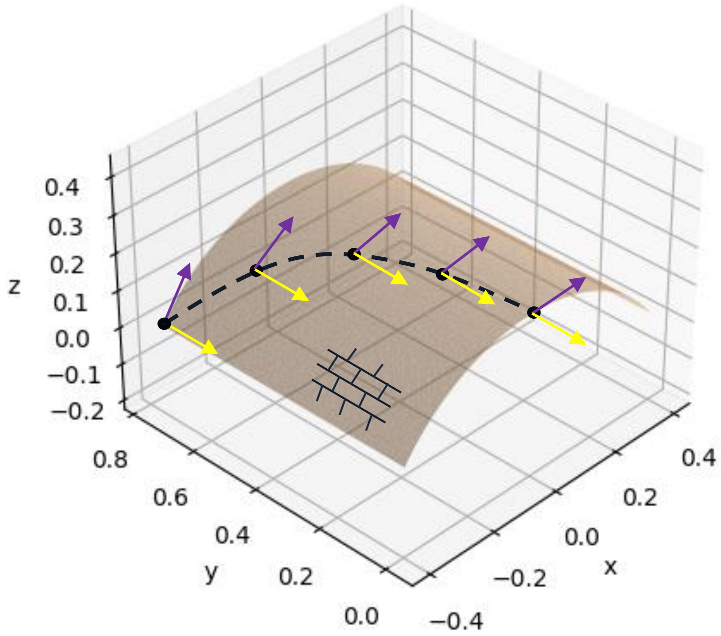
Approach 1 - piecewise approximation:

- Planar DLO problems are studied extensively
- Easy to model complex geometries
- Potentially inaccurate due to approximation



Approach 2 – parametric geometry representation

- Accurate geometry, potentially better (more conservative) upper-bound results
- More difficult mathematics
- Requires further validation



Introduction to DLO

Introduction to DLO

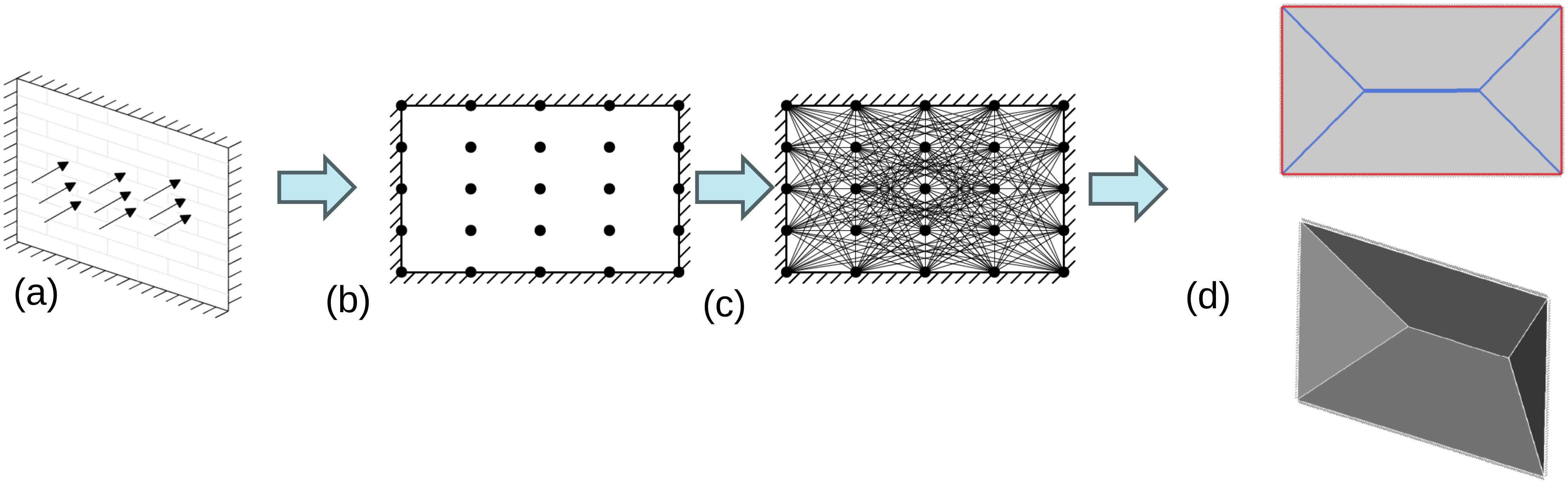
Standard DLO method

For a given (a) structural problem:

(b) discretise with nodes

(c) interconnect nodes with possible discontinuities

(d) find **critical yield-line pattern** and **load factor** through **optimization**



Introduction to DLO

Formulation

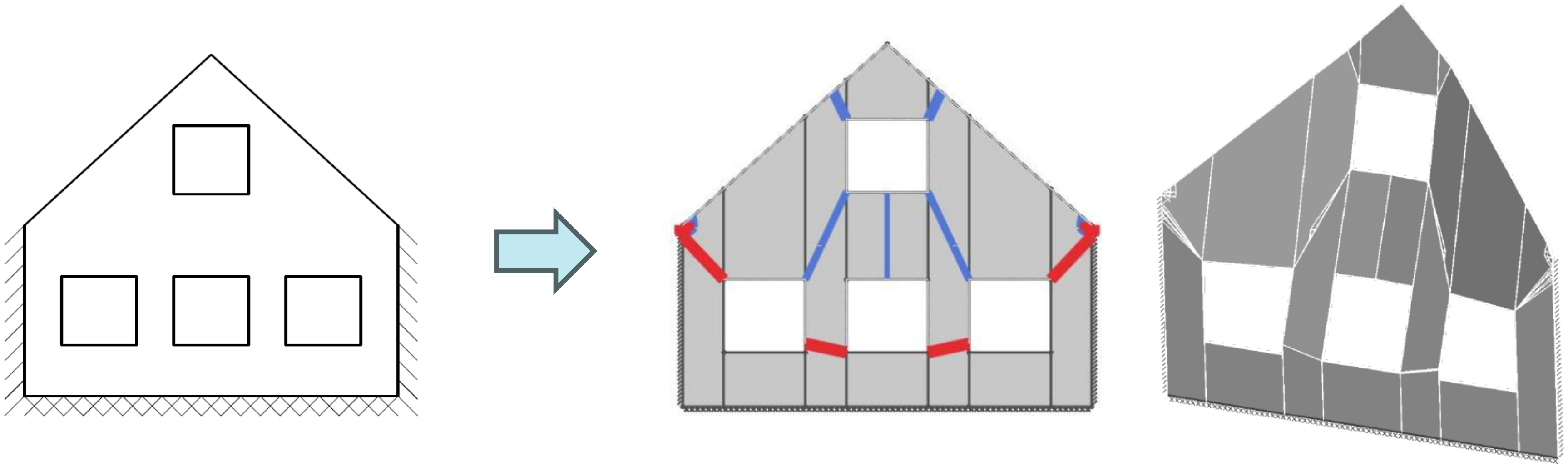
$$
\begin{aligned}
\text{minimize} \quad & \lambda \mathbf{f}_L^T \mathbf{d} = -\mathbf{f}_D^T \mathbf{d} + \mathbf{g}^T \mathbf{p} && \text{Principle of virtual work} \\
\text{subject to} \quad & \mathbf{B}\mathbf{d} = \mathbf{0} && \text{Compatibility} \\
& \mathbf{N}\mathbf{p} - \mathbf{d} = \mathbf{0} && \text{Flow rule} \\
& \mathbf{f}_L^T \mathbf{d} = 1 && \text{Unit live-load work} \\
& \mathbf{p} \geq 0 && \text{Non-negative plastic mutlipliers}
\end{aligned}
$$

Advantages:

- Simple linear programming problem
- Efficient
- Accurate representation of failure modes

Introduction to DLO

Sample DLO application inspired by a real case



Drawbacks
- actual masonry texture is ignored
- not conceived for curved geometries

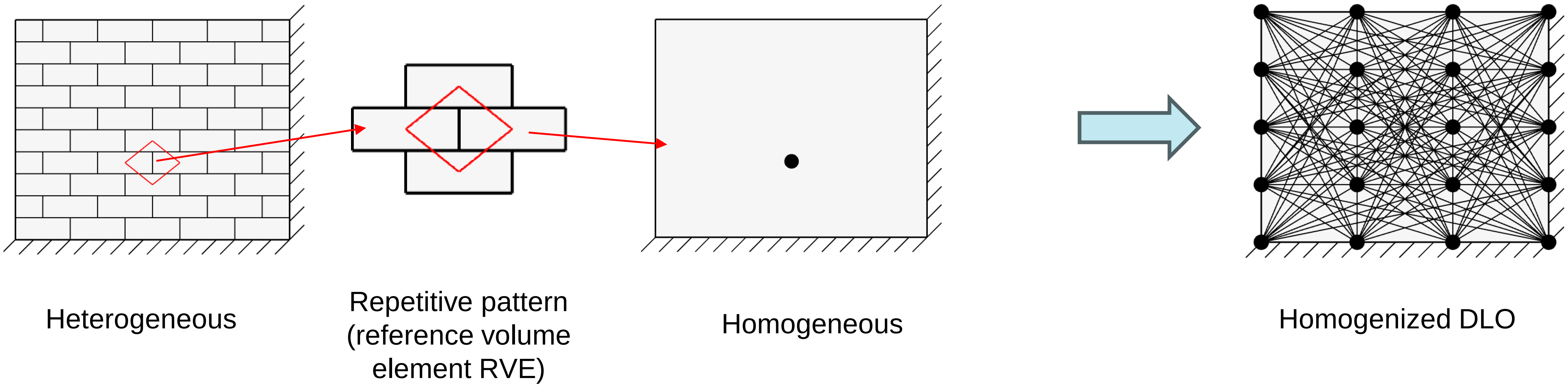
…still not suited for masonry bridges

Required:
(1) homogenization
(2) parametric geometry

Key concepts

Key concepts: (1) homogenization

The theory of **homogenization** is here used to move from an heterogeneous material (the actual masonry texture) to an equivalent homogenous one.



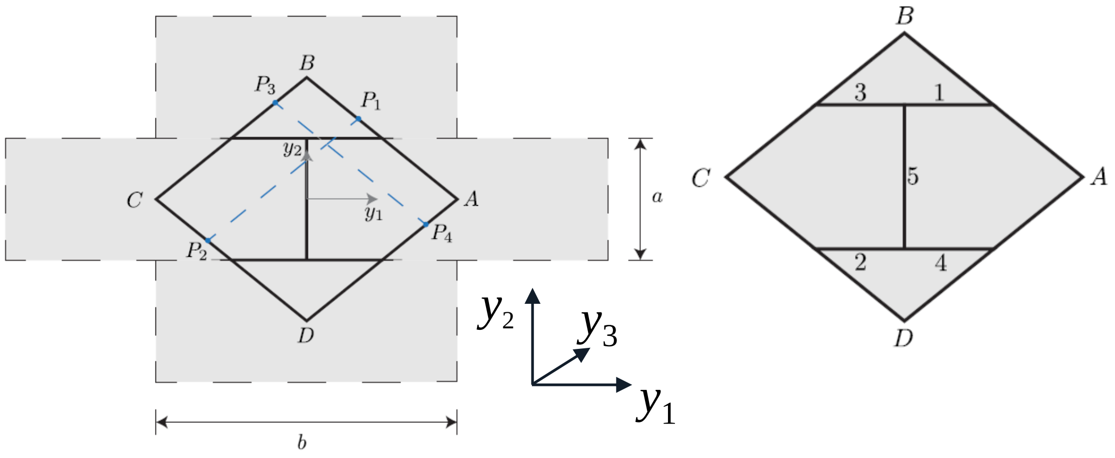
DLO can be applied on the homogeneous material, provided that:

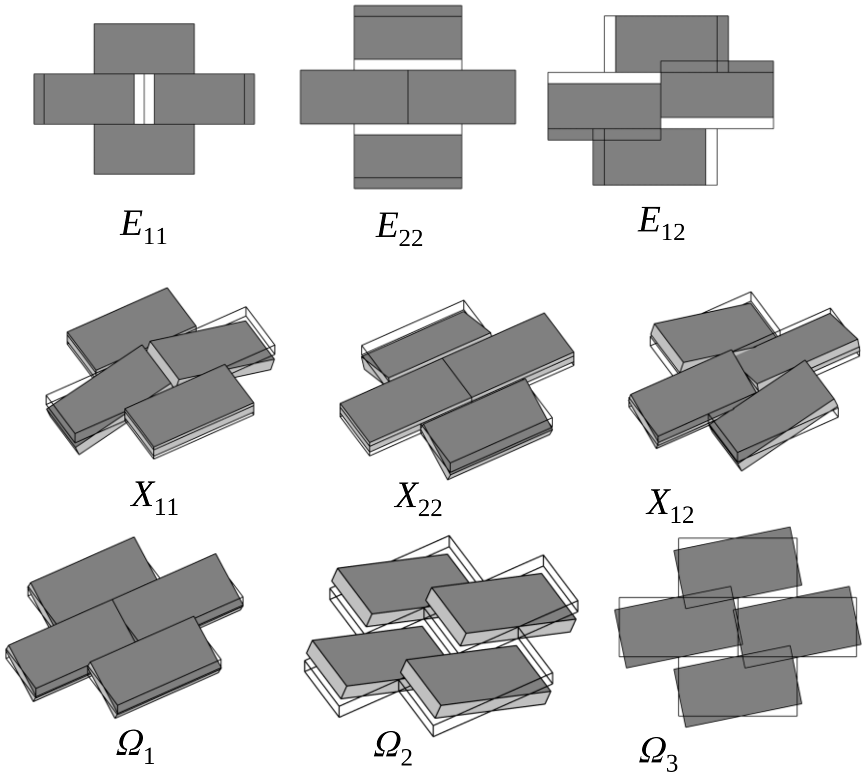
- equivalent energy dissipation,
- models all potential failure modes,
- flow rule is satisfied.

Key concepts: (1) homogenization

Main steps:

- identify RVE and microscopic variables (strains, curvatures and rotations):



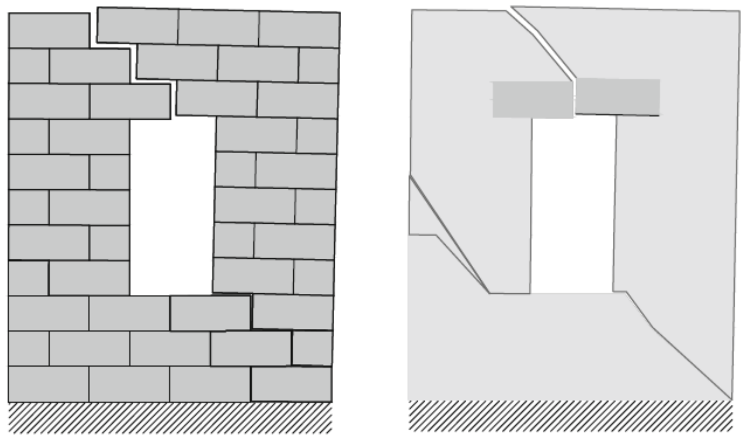


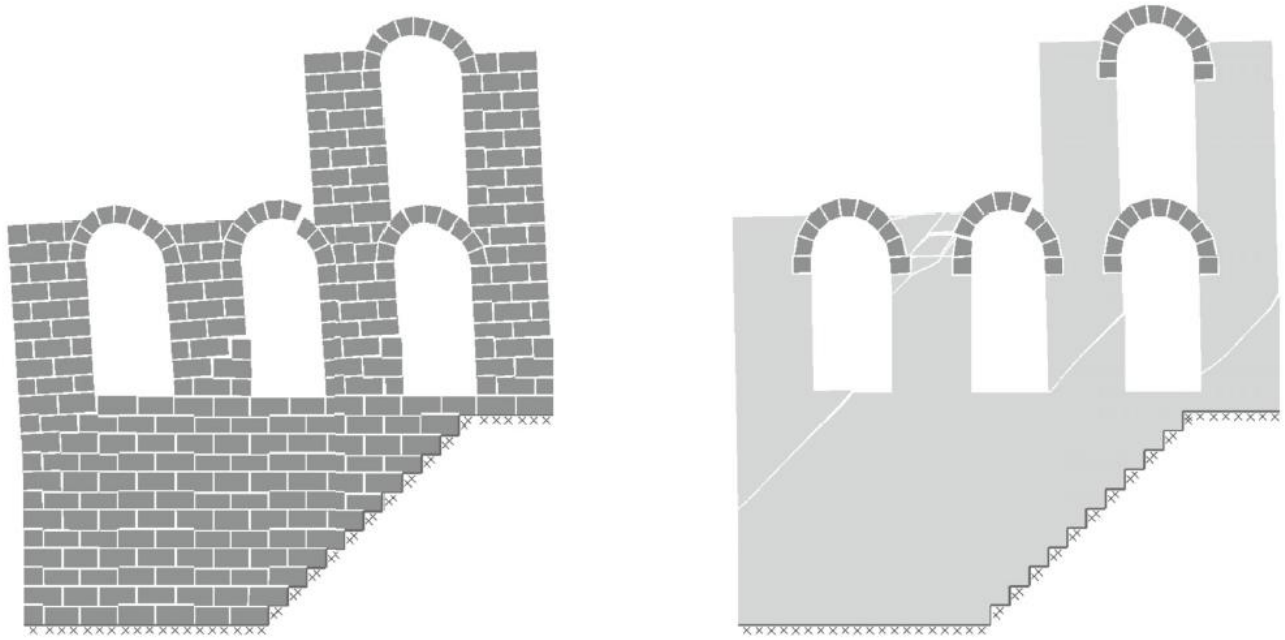
- evaluate the energy with reference to the microscopic variables

- derive the constraints related to the associated flow rule (no compenetration, proper representation of shear)

Key concepts: (1) homogenization

Homogenization DLO available for in-plane loaded masonry structures



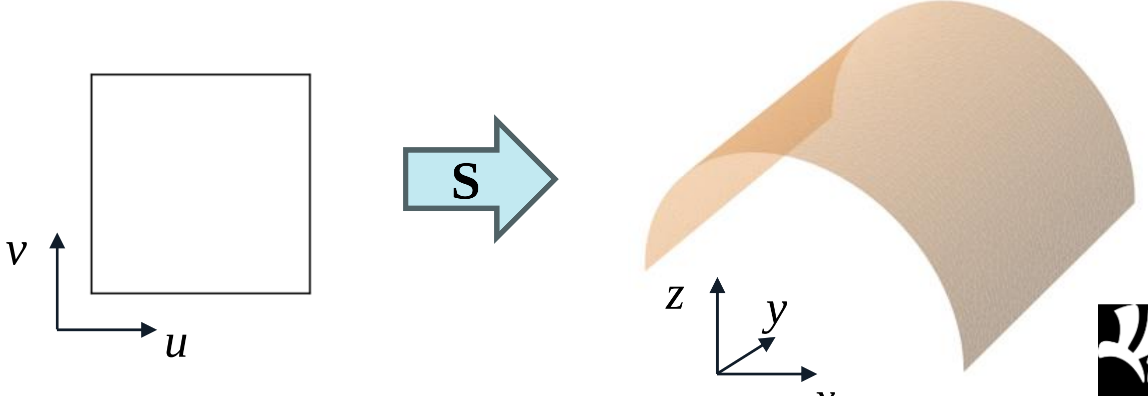


However, the case of masonry arch bridges requires a generalised geometrical representation.

Key concepts: (2) parametric geometry

A geometrical representation via **parametric surfaces** $\mathbf{S}(u, v)$ allows an easy modelling of curved shapes.

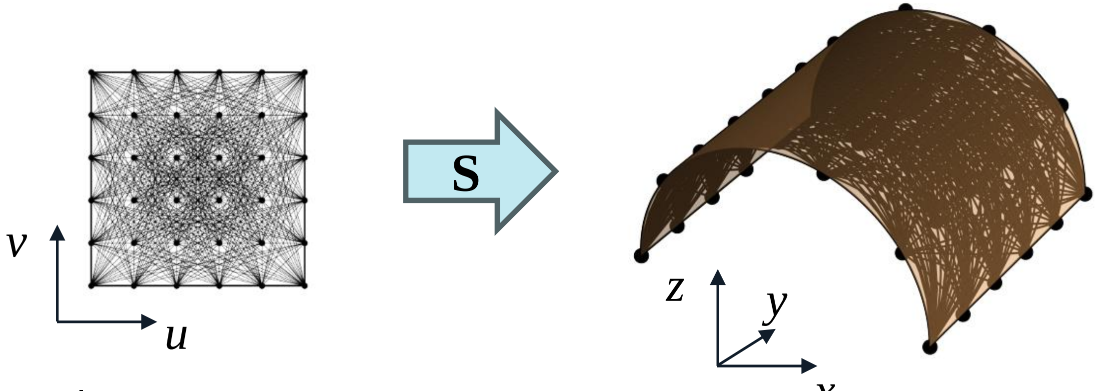
$$\mathbf{S} : [0,1]^2 \rightarrow \mathbb{R}^3, \mathbf{S}(u,v) = \begin{bmatrix} S_x(u,v) \\ S_y(u,v) \\ S_z(u,v) \end{bmatrix}$$



2D parametric domain

Surface in the 3D parametric domain

In DLO:
- nodes defined in the parametric domain
- 3D layout is automatically generated

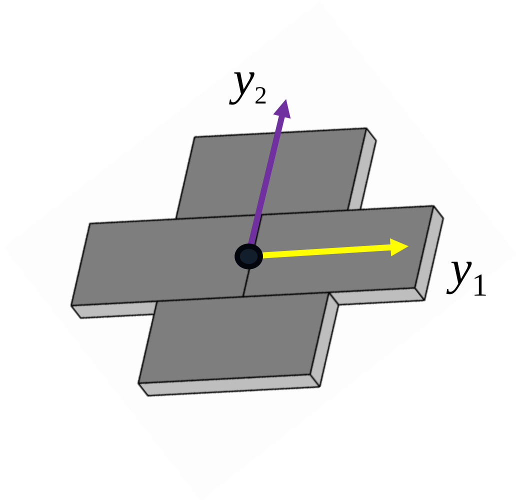


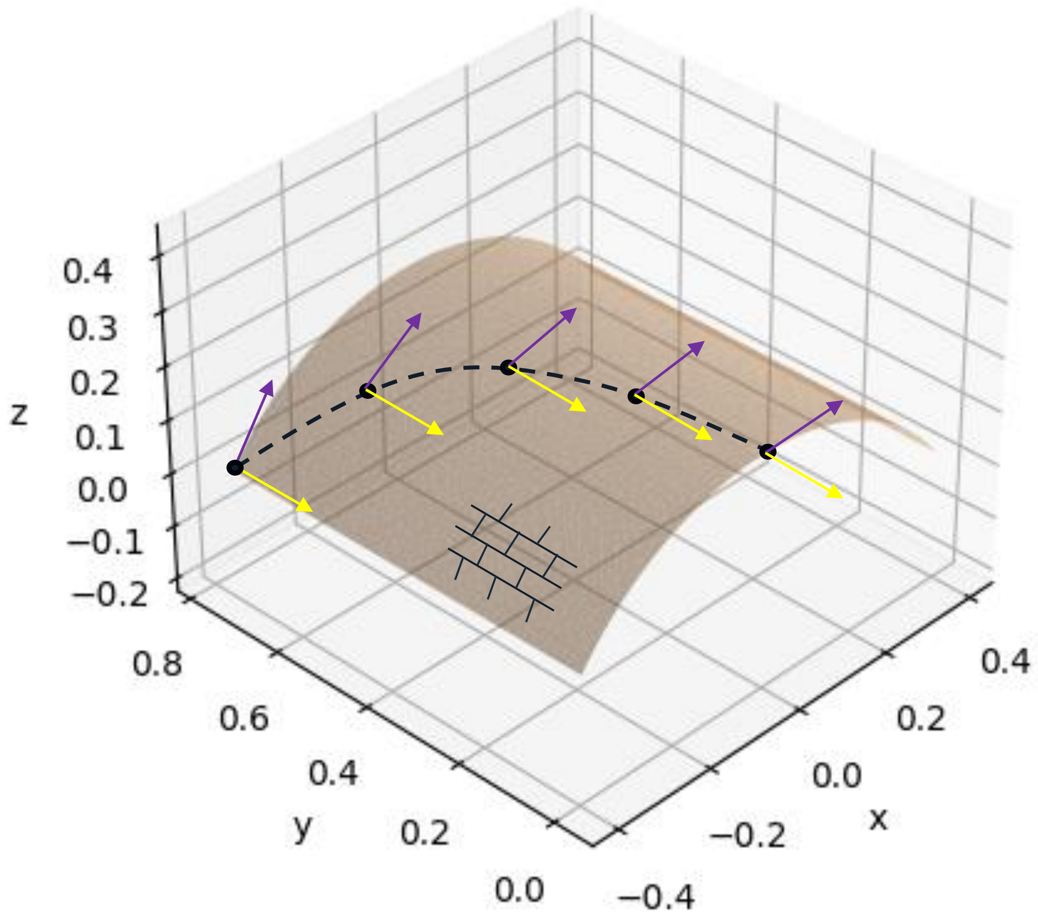
2D layout

3D curved layout

Key concepts: (2) parametric geometry

Some points to be mentioned about curved discontinuities:

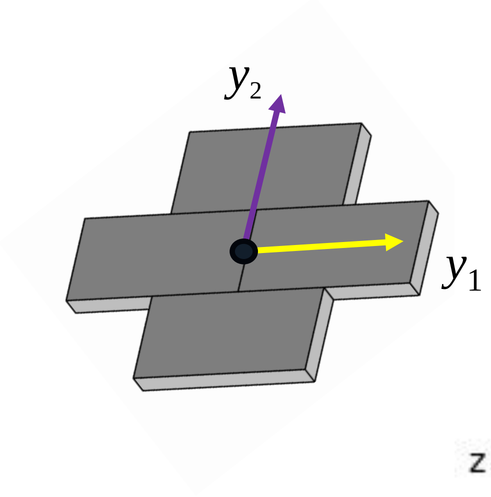


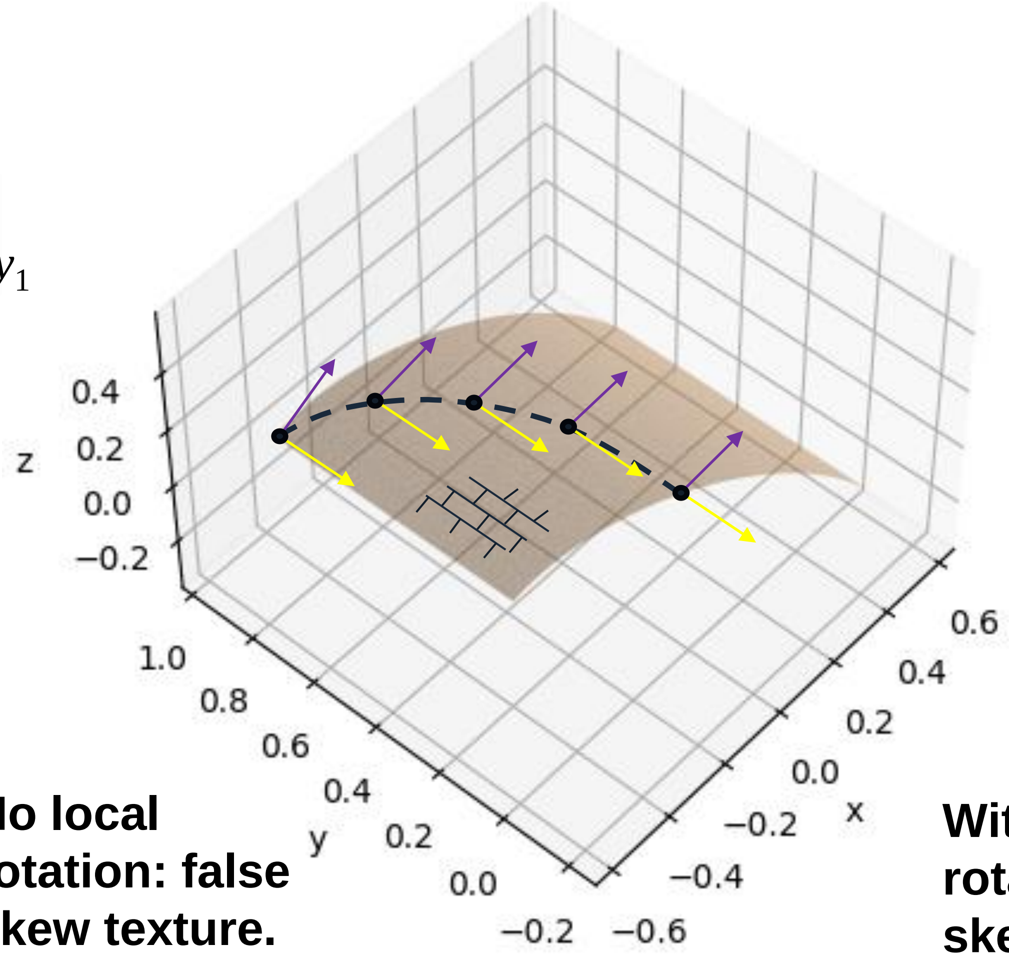


- The flow rule must be imposed along a certain number of collocation points.
- Local axes define the **texture orientation**.
- Local rotation can be required.

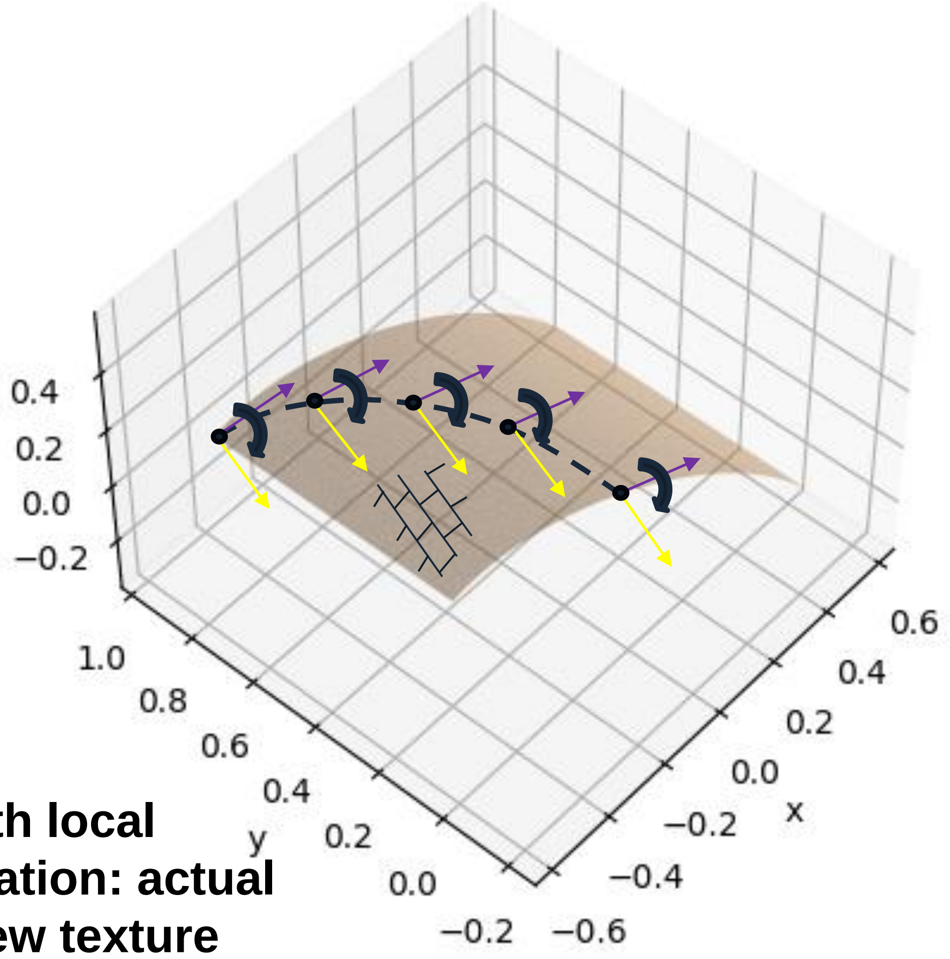
Key concepts: (2) parametric geometry

Some points to be mentioned about curved discontinuities:





No local rotation: false skew texture.



With local rotation: actual skew texture

Homogenized DLO formulation

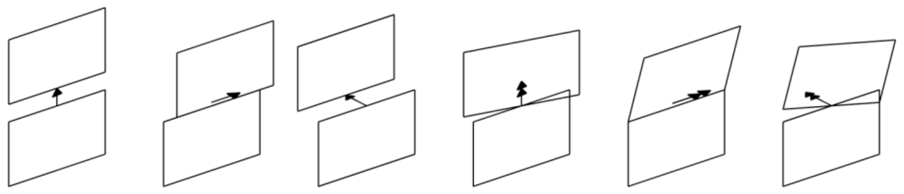
Homogenized DLO formulation

The multi-scale DLO formulation is obtained by combining DLO with homogenization and parametric geometry:

$$\begin{aligned} \text{minimize} \quad & \lambda \mathbf{f}_L^T \mathbf{d} = -\mathbf{f}_D^T \mathbf{d} + \sum_i^m l_i \mathbf{w}^T \mathbf{e} && \text{Energy equivalence} \\ \text{subjected to} \quad & \mathbf{B}\mathbf{d} = \mathbf{0} && \text{Compatibility} \\ & \mathbf{f}_L^T \mathbf{d} = 1 && \text{Unitary live load work} \\ & \mathbf{e} = \mathbf{T}\mathbf{d} && \text{Coupling between strains and displacements} \\ & \mathbf{P}(\mathbf{e}) <= \mathbf{0} && \text{Flow rule} \end{aligned}$$

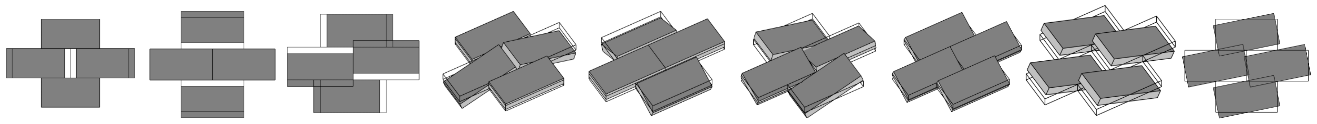
Displacement jumps (for each discontinuity):

$$\mathbf{d}_i = \begin{bmatrix} d_n & d_s & d_t & r_n & r_s & r_t \end{bmatrix}^T$$



Strains at the RVE (for each collocation point) :

$$\mathbf{e}_k = \begin{bmatrix} E_{11} & E_{22} & E_{12} & X_{11} & X_{22} & X_{12} & \Omega_1 & \Omega_2 & \Omega_3 \end{bmatrix}^T$$



Homogenized DLO formulation

Simple workflow

Input

Geometry
Load
Material
Node density

Pre-processing

Define matrices

Processing

Solve optmization problem

Post-processing

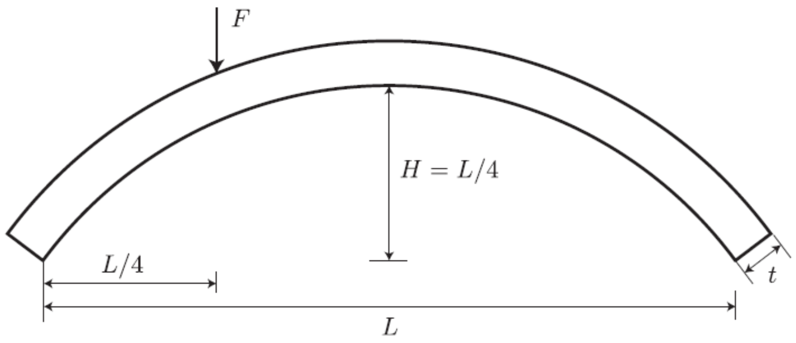
Extract solution

Output

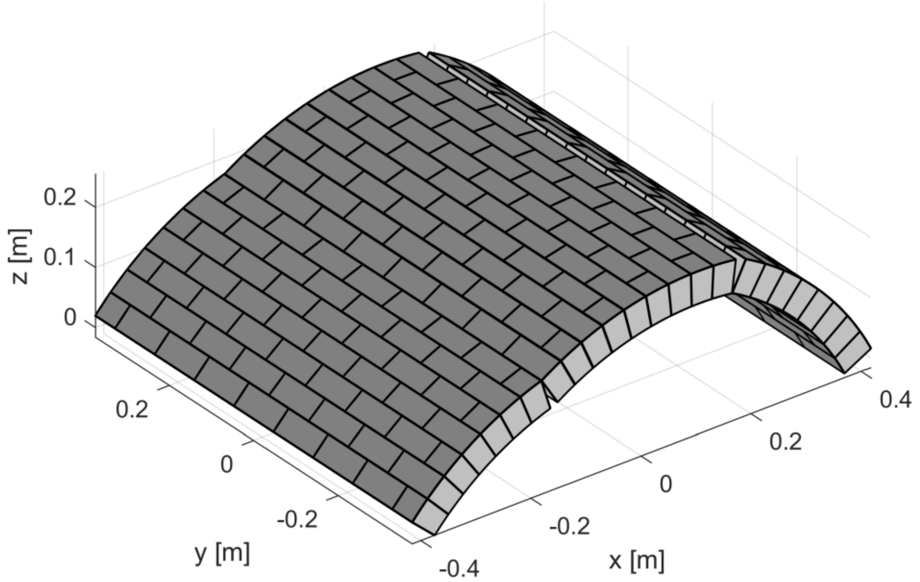
Collapse load
Critical crack pattern

Numerical examples

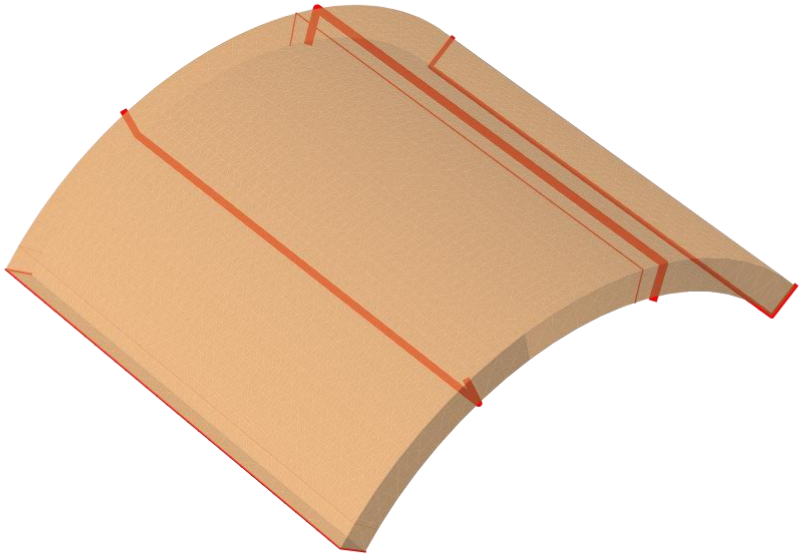
Numerical examples: square arch under line load



L = 750 mm
t = 54 mm
$w = L$
b/a = 2.47
γ = 23.6 kN/m^3
c = 0
μ = 0.75

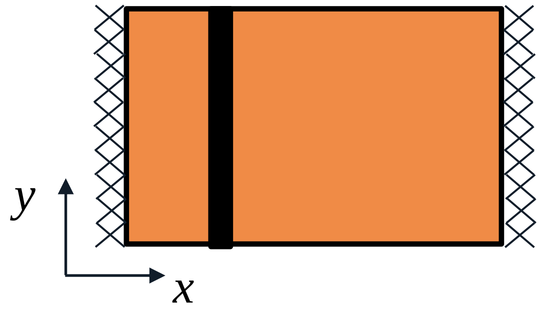


Collapse load: 0.79 kN

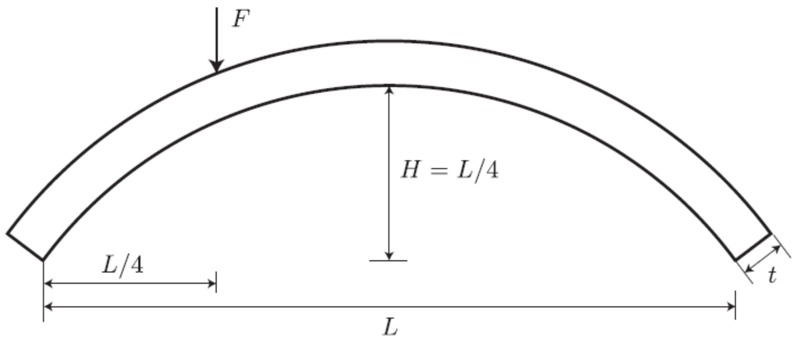


Collapse load: 0.785 kN

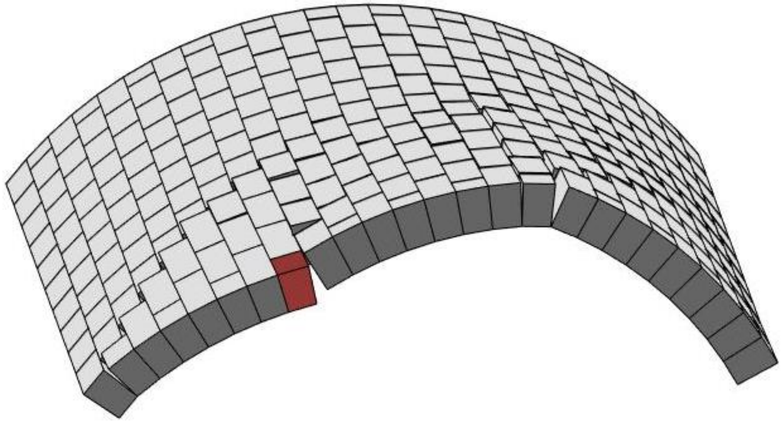
Load position



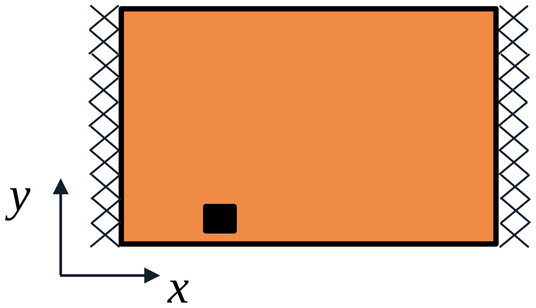
Numerical examples: square arch under point load



L = 750 mm
t = 54 mm
w = L
b/a = 2.04
γ = 23.6 kN/m^3
c = 0
μ = 0.75



Load position



Collapse load: 0.68 kN

Collapse load: 0.49 kN

Conclusions

Conclusions

- The efficacy of three fast running tools are being explored to model 3D masonry arch bridge behaviour (rigid block, physics engine and DLO)
- In the case of the DLO-based tool:
  - DLO is a fast and potentially highly accurate limit analysis tool
  - Masonry arch bridges can be modeled via homogenized DLO
  - Parametric geometry allows accurate geometrical representation
  - 3D failure mechanisms appear to broadly match rigid blocks analysis solutions